

A PARALLEL QR-FACTORIZATION/SOLVER OF QUASISEPARABLE MATRICES*

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Abstract. This manuscript focuses on the development of a parallel QR -factorization of structured rank matrices, which can then be used for solving systems of equations. First, we will prove the existence of two types of Givens transformations, named rank decreasing and rank expanding Givens transformations. Combining these two types of Givens transformations leads to different patterns for annihilating the lower triangular part of structured rank matrices. How to obtain different annihilation patterns, for computing the upper triangular factor R , such as the \vee and \wedge pattern will be investigated. Another pattern, namely the \times -pattern, will be used for computing the QR -factorization in a parallel way.

As an example of such a parallel QR -factorization, we will implement it for a quasiseparable matrix. This factorization can be run on 2 processors, with one step of intermediate communication in which one row needs to be sent from one processor to the other and back. Another example, showing how to deduce a parallel QR -factorization for a more general rank structure will also be discussed.

Numerical experiments are included for demonstrating the accuracy and speed of this parallel algorithm w.r.t. the existing factorization of quasiseparable matrices. Also some numerical experiments on solving systems of equations using this approach will be given.

Key words. parallel QR -factorization, structured rank matrices, quasiseparable matrix

AMS subject classifications. 65F05

*Received October 19, 2006. Accepted for publication March 11, 2008. Published online on June 26, 2008. Recommended by V. Olshevsky. The research was partially supported by the Research Council K.U.Leuven, project OT/05/40 (Large rank structured matrix computations), Center of Excellence: Optimization in Engineering, by the Fund for Scientific Research–Flanders (Belgium), Iterative methods in numerical Linear Algebra, G.0455.0 (RHPH: Riemann-Hilbert problems, random matrices and Padé-Hermite approximation), G.0423.05 (RAM: Rational modelling: optimal conditioning and stable algorithms), and by the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister’s Office for Science, Technology and Culture, project IUAP V-22 (Dynamical Systems and Control: Computation, Identification & Modelling). The first author has a grant of “Postdoctoraal Onderzoeker” from the Fund of Scientific Research Flanders (FWO-Vlaanderen). The research of the third author was partially supported by MIUR, grant number 2004015437, by the short term mobility program, Consiglio Nazionale delle Ricerche and by VII Programma Esecutivo di Collaborazione Scientifica Italia–Comunità Francese del Belgio, 2005–2006. The scientific responsibility rests with the authors.

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