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A REDUCED BASIS METHOD FOR EVOLUTION SCHEMES WITH PARAMETER-DEPENDENT EXPLICIT OPERATORS*

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Abstract. During the last decades, reduced basis (RB) methods have been developed to a wide methodology for model reduction of problems that are governed by parametrized partial differential equations (P^2DEs). In particular equations of elliptic and parabolic type for linear, low degree polynomial or monotonic nonlinearities have been treated successfully by RB methods using finite element schemes. Due to the characteristic offline-online decomposition, the reduced models often become suitable for a multi-query or real-time setting, where simulation results, such as field-variables or output estimates, can be approximated reliably and rapidly for varying parameters. In the current study, we address a certain class of time-dependent evolution schemes with explicit discretization operators that are arbitrarily parameter dependent. We extend the RB methodology to these cases by applying the *empirical interpolation* method to localized discretization operators. The main technical ingredients are: (i) generation of a *collateral reduced basis* modelling the effects of the discretization operator under parameter variations in the offline-phase and (ii) an online simulation scheme based on a numerical subgrid and localized evaluations of the evolution operator. We formulate an a-posteriori error estimator for quantification of the resulting reduced simulation error. Numerical experiments on a parametrized convection problem, discretized with a finite volume scheme, demonstrate the applicability of the model reduction technique. We obtain a parametrized reduced model, which enables parameter variation with fast simulation response. We quantify the computational gain with respect to the non-reduced model and investigate the error convergence.

Key words. model reduction, reduced basis methods, parameter dependent explicit operators, empirical interpolation, a-posteriori error estimates

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