

Electronic Transactions on Numerical Analysis

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Contents

- 1** Identifying the magnetic permeability in multi-frequency EM data inversion.
Gian Piero Deidda, Patricia Díaz de Alba, and Giuseppe Rodriguez.

Abstract.

Electromagnetic induction surveys are among the most popular techniques for non-destructive investigation of soil properties in order to detect the presence of either ground inhomogeneities or of particular substances. In this paper we develop a regularized algorithm for the inversion of a nonlinear mathematical model well established in applied geophysics, starting from noisy electromagnetic data collected by varying both the height of the measuring device with respect to the ground level and its operating frequency. Assuming the conductivity to be known in advance, we focus on the determination of the magnetic permeability of the soil with respect to depth, and give the analytical expression of the Jacobian matrix of the forward model, which is indispensable for the application of the inversion algorithm. Finally, numerical experiments on synthetic data sets illustrate the effectiveness of the method.

Key Words.

Regularization, nonlinear inverse problems, electromagnetic induction

AMS Subject Classifications.

65F22, 65H10, 65R32, 86A22

- 18** A block Arnoldi based method for the solution of the Sylvester-observer equation.
L. Elbouyahyaoui, M. Heyouni, K. Jbilou, and A. Messaoudi.

Abstract.

This paper describes a new block method for solving multi-input Sylvester-observer equations that arise in the construction of the well-known Luenberger observer. The proposed method is based on the block Arnoldi process and generalizes to the multi-input case, the method proposed by Datta and Saad for the single input Sylvester-observer equation. We give new algebraic properties and show how to construct the Luenberger observer by solving a special large-scale Sylvester equation for which two unknown matrices are to be computed. The numerical tests show that the proposed approach is effective and can be used for large-scale Luenberger observer problems.

Key Words.

Block Arnoldi process, Luenberger-observer, Sylvester equation

AMS Subject Classifications.

65F10

- 37 Any admissible harmonic Ritz value set is possible for GMRES.
Kui Du, Jurjen Duintjer Tebbens, and Gérard Meurant.

Abstract.

It is known that the harmonic Ritz values are the zeros of the GMRES residual polynomials. In this paper we show that any decreasing GMRES residual norm history is possible with any prescribed finite harmonic Ritz values in every iteration of the GMRES process. In addition, we characterize the admissible harmonic Ritz values when GMRES stagnates during some iterations, and show that with any prescribed, in this sense admissible harmonic Ritz values, any non-increasing GMRES residual norm history is possible. We also present a parameterization of the class of matrices and right-hand sides yielding prescribed GMRES residual norms and prescribed admissible harmonic Ritz values.

Key Words.

Ritz values, harmonic Ritz values, GMRES convergence, prescribed residual norms, FOM convergence

AMS Subject Classifications.

65F15, 65F10

- 57 Incremental computation of block triangular matrix exponentials with application to option pricing.
Daniel Kressner, Robert Luce, and Francesco Statti.

Abstract.

We study the problem of computing the matrix exponential of a block triangular matrix in a peculiar way: block column by block column, from left to right. The need for such an evaluation scheme arises naturally in the context of option pricing in polynomial diffusion models. In this setting, a discretization process produces a sequence of nested block triangular matrices, and their exponentials are to be computed at each stage until a dynamically evaluated criterion allows to stop. Our algorithm is based on scaling and squaring. By carefully reusing certain intermediate quantities from one step to the next, we can efficiently compute such a sequence of matrix exponentials.

Key Words.

matrix exponential, block triangular matrix, polynomial diffusion models, option pricing

AMS Subject Classifications.

15A16, 65F60, 91G20

- 73 On generalized iterated Tikhonov regularization with operator-dependent seminorms.
Davide Bianchi and Marco Donatelli.

Abstract.

We investigate the recently introduced Tikhonov regularization filters with penalty terms having seminorms that depend on the operator itself. Exploiting the singular value decomposition of the operator, we provide optimal order conditions, smoothing properties, and a general condition (with a minor condition of the seminorm) for

the saturation level. Moreover, we introduce and analyze both stationary and non-stationary iterative counterparts of the generalized Tikhonov method with operator-dependent seminorms. We establish their convergence rate under conditions affecting only the iteration parameters, proving that they overcome the saturation result. Finally, some selected numerical results confirm the effectiveness of the proposed regularization filters.

Key Words.

ill-posed problems, fractional Tikhonov regularization, iterated Tikhonov, filter functions

AMS Subject Classifications.

65F22, 47A52, 65R32

- 100** Block Krylov subspace methods for functions of matrices.
Andreas Frommer, Kathryn Lund, and Daniel B. Szyld.

Abstract.

A variety of block Krylov subspace methods have been successfully developed for linear systems and matrix equations. The application of block Krylov methods to compute matrix functions is, however, less established, despite the growing prevalence of matrix functions in scientific computing. Of particular importance is the evaluation of a matrix function on not just one but multiple vectors. The main contribution of this paper is a class of efficient block Krylov subspace methods tailored precisely to this task. With the full orthogonalization method (FOM) for linear systems forming the backbone of our theory, the resulting methods are referred to as $B(\text{FOM})^2$: block FOM for functions of matrices.

Many other important results are obtained in the process of developing these new methods. Matrix-valued inner products are used to construct a general framework for block Krylov subspaces that encompasses already established results in the literature. Convergence bounds for $B(\text{FOM})^2$ are proven for Stieltjes functions applied to a class of matrices which are self-adjoint and positive definite with respect to the matrix-valued inner product. A detailed algorithm for $B(\text{FOM})^2$ with restarts is developed, whose efficiency is based on a recursive expression for the error, which is also used to update the solution. Numerical experiments demonstrate the power and versatility of this new class of methods for a variety of matrix-valued inner products, functions, and matrices.

Key Words.

matrix functions, restarted Krylov subspace methods, block Krylov subspace methods, global methods

AMS Subject Classifications.

65F60, 65F50, 65F10, 65F30

- 127** An optimal Q-OR Krylov subspace method for solving linear systems.
G rard Meurant.

Abstract.

Today the most popular iterative methods for solving nonsymmetric linear systems are Krylov methods. In this paper we show how to construct a non-orthogonal basis of the Krylov subspace such that the quasi-orthogonal residual (Q-OR) Krylov method using this basis yields the same residual norms as GMRES up to the final

stagnation phase, provided GMRES is not stagnating. In many examples this new Krylov method gives a better maximum attainable accuracy than GMRES with a modified Gram-Schmidt (MGS) implementation. Even though the number of floating point operations per iteration is larger than for GMRES, the optimal Q-OR method offers more potential for parallelism than GMRES with MGS.

Key Words.

linear systems, Krylov methods, Q-OR algorithm

AMS Subject Classifications.

65F10

- 153 Weighted Golub-Kahan-Lanczos bidiagonalization algorithms.

Hong-Xiu Zhong and Hongguo Xu.

Abstract.

We present weighted Golub-Kahan-Lanczos algorithms. We demonstrate their applications to the eigenvalue problem of a product of two symmetric positive definite matrices and an eigenvalue problem for the linear response problem. A convergence analysis is provided and numerical test results are reported. As another application we make a connection between the proposed algorithms and the preconditioned conjugate gradient (PCG) method.

Key Words.

weighted Golub-Kahan-Lanczos bidiagonalization algorithm, eigenvalue, eigenvector, Ritz value, Ritz vector, linear response eigenvalue problem, Krylov subspace, bidiagonal matrices

AMS Subject Classifications.

65F15, 15A18

- 179 Vector estimates for $f(\mathbf{A})\mathbf{b}$ via extrapolation.

Marilena Mitrouli and Paraskevi Roupa.

Abstract.

Let $\mathbf{A} \in \mathbb{R}^{p \times p}$ be a diagonalizable matrix and f a smooth function. We are interested in the problem of approximating the action of $f(\mathbf{A})$ on a vector $\mathbf{b} \in \mathbb{R}^p$, i.e., $f(\mathbf{A})\mathbf{b}$, without explicitly computing the matrix $f(\mathbf{A})$. In the present work, we derive families of one-term, two-term, and three-term inexpensive approximations to the quantity $f(\mathbf{A})\mathbf{b}$ via an extrapolation procedure. For a given diagonalizable matrix \mathbf{A} , the proposed families of vector estimates allow us to approximate the form $\mathbf{W}^T f(\mathbf{A})\mathbf{U}$, for any matrices $\mathbf{W}, \mathbf{U} \in \mathbb{R}^{p \times m}$, $1 \leq m \ll p$, not necessarily biorthogonal. We present several numerical examples to illustrate the effectiveness of our method for several functions f for both the quantity $f(\mathbf{A})\mathbf{b}$ and the form $\mathbf{W}^T f(\mathbf{A})\mathbf{U}$.

Key Words.

$f(\mathbf{A})\mathbf{b}$, vector estimates, vector moments, extrapolation, diagonalizable matrices

AMS Subject Classifications.

65F15, 65F30, 65F60, 65B05, 15A18

- 197 Enhanced matrix function approximation.
Nasim Eshghi, Lothar Reichel, and Miodrag M. Spalević.

Abstract.

Matrix functions of the form $f(\mathbf{A})\mathbf{v}$, where \mathbf{A} is a large symmetric matrix, f is a function, and $\mathbf{v} \neq \mathbf{0}$ is a vector, are commonly approximated by first applying a few, say n , steps of the symmetric Lanczos process to \mathbf{A} with the initial vector \mathbf{v} in order to determine an orthogonal section of \mathbf{A} . The latter is represented by a (small) $n \times n$ tridiagonal matrix to which f is applied. This approach uses the n first Lanczos vectors provided by the Lanczos process. However, n steps of the Lanczos process yield $n+1$ Lanczos vectors. This paper discusses how the $(n+1)$ st Lanczos vector can be used to improve the quality of the computed approximation of $f(\mathbf{A})\mathbf{v}$. Also the approximation of expressions of the form $\mathbf{v}^T f(\mathbf{A})\mathbf{v}$ is considered.

Key Words.

matrix function, symmetric Lanczos process, Gauss quadrature

AMS Subject Classifications.

65D32, 65F10, 65F60

- 206 Varying the s in your s -step GMRES.
David Imberti and Jocelyne Erhel.

Abstract.

Krylov subspace methods are commonly used iterative methods for solving large sparse linear systems. However, they suffer from communication bottlenecks on parallel computers. Therefore, s -step methods have been developed, where the Krylov subspace is built block by block so that s matrix-vector multiplications can be done before orthonormalizing the block. Then Communication-Avoiding algorithms can be used for both kernels. This paper introduces a new variation on the s -step GMRES method in order to reduce the number of iterations necessary to ensure convergence with a small overhead in the number of communications. Namely, we develop an s -step GMRES algorithm, where the block size is variable and increases gradually. Our numerical experiments show a good agreement with our analysis of condition numbers and demonstrate the efficiency of our variable s -step approach.

Key Words.

Communication-Avoiding, s -step Krylov subspace method, GMRES algorithm, variable s -step

AMS Subject Classifications.

65F10, 65N22